N* Masses from an Anisotropic Lattice QCD Action

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We report N* masses in the spin-3/2 sector from a highly-improved anisotropic action. States with both positive and negative parity are isolated via a parity projection method. The extent to which spin projection is needed is examined. The gross features of the splittings from the nucleon ground state show a trend consistent with experimental results at the quark masses explored.

1. Introduction

Lattice QCD plays an important role in understanding the N* spectrum. One can systematically study the spectrum sector by sector, with the ability to dial the quark masses, and dissect the degrees of freedom. The rich structure of the excited baryon spectrum, as tabulated by the particle data group [1], provides a fertile ground for exploring how the internal degrees of freedom in the nucleon are excited and how QCD works in a wider context. One outstanding example is the parity splitting pattern in the low-lying N* spectrum. The splittings are a direct manifestation of spontaneous chiral symmetry breaking of QCD. Without it, QCD predicts parity doubling in the baryon spectrum. The experimental effort on excited baryons has intensified in recent years at JLab and other accelerators, generating renewed debate on how well these states are known. The star-rating system on the observed states is a reflection of the current situation.

Given that state-of-the-art lattice QCD simulations have produced a ground-state spectrum that is very close to the observed values [2], it is important to extend the success beyond the ground states. There exist already a number of lattice studies of the N* spectrum [3–8], focusing mostly on the spin-1/2 sector. All established a clear splitting from the ground state. In this study, we explore the possibility of calculating the excited baryon states in the spin-3/2 sector with isospin 1/2.

2. Method

We consider the following interpolating field with the quantum numbers $I(J^P) = \frac{1}{2}\left(\frac{3}{2}^+\right)$ [9, 10],

$$\chi_\mu = \epsilon^{abc}\left(u_a^T C\gamma_5\gamma_\rho d_b\right)\left(g^{\mu\rho} - \frac{1}{4}\gamma^\mu\gamma^\rho\right)\gamma_5 u_c. \quad (1)$$

Despite having an explicit parity by construction, the interpolating field couples to both positive and negative parity states. A parity projection is needed to separate the two. In the large Euclidean time limit, the correlator with Dirichlet boundary condition in the time direction and zero spatial momentum becomes

$$G_{\mu\nu}(t) = \sum_{\mathbf{x}} < 0|\chi_\mu(x)\,\overline{\chi}_\nu(0)|0 > \quad (2)$$

$$= f_{\mu\nu}\left[\lambda_+^2\frac{\gamma_4+1}{2}e^{-M_+ t} + \lambda_-^2\frac{-\gamma_4+1}{2}e^{-M_- t}\right] \quad (3)$$

where $f_{\mu\nu}$ is a function common to both terms. The relative sign in front of γ_4 provides the solution: by taking the trace of $G_{\mu\nu}(t)$ with $(1\pm\gamma_4)/4$, one can isolate M_+ and M_-, respectively.

It is well-known that a spin-3/2 interpolating field couples to both spin-3/2 and spin-1/2 states. A spin projection can be used to isolate the individual contributions in the correlation function $G_{\mu\nu}$. Using the spin-3/2 projection operator [11],

$$P^{3/2}_{\mu\nu} = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}(\gamma\cdot p\gamma_\mu p_\nu + p_\mu\gamma_\nu\gamma\cdot p) \quad (4)$$

the spin-3/2 part can be projected out by

$$G^{3/2}_{\mu\nu}(t) = \sum_{\lambda=1}^{4} G_{\mu\lambda}(t) P^{3/2}_{\lambda\nu}, \quad (5)$$

while the spin-1/2 part by

$$G^{1/2}_{\mu\nu}(t) = \sum_{\lambda=1}^{4} G_{\mu\lambda}(t)(1 - P^{3/2}_{\lambda\nu}). \quad (6)$$

Obviously, they satisfy the relation

$$G_{\mu\nu}(t) = G^{1/2}_{\mu\nu}(t) + G^{3/2}_{\mu\nu}(t). \quad (7)$$

The anisotropic gauge action of [12], and the anisotropic D234 quark action of [13] are used. Both have tadpole-improved tree-level coefficients. A $10^3 \times 30$ lattice with $a_s \approx 0.24$ fm and anisotropy $\xi = a_s/a_t = 3$ is used. In all, 100 configurations are analyzed. On each configuration 9 quark propagators are computed using a multi-mass solver, with quark masses ranging from approximately 780 to 90 MeV. The corresponding mass ratio π/ρ is from 0.95 to 0.65. A gauge-invariant gaussian-smeared source is used. The source is located at $(x, y, z, t) = (2, 2, 2, 2)$.

Figure 1 presents results for the correlation function in the positive-parity nucleon channel at the smallest quark mass considered. Spin projection reveals two different exponentials from the spin-3/2 and spin-1/2 parts, with the spin-3/2 state being heavier than the spin-1/2 one (a steeper fall-off), in agreement with the ordering in experiment. The expected relation in Eq. (7) is indeed satisfied numerically, providing a nontrivial check of the calculation. A further check of the calculation is provided by the fact that the mass extracted from $G^{1/2}_{\mu\nu}(t)$ is degenerate with that from the conventional $G(t)$ for the nucleon ground state using the interpolating field $\chi = \epsilon^{abc}\left(u_a^T C\gamma_5 d_b\right) u_c$. One would get a false signal from the dominant spin-1/2 state without spin projection. The large error bars indicate sensitive cancellations in the projection procedure.

Figure 2 shows the similar plot in the negative-parity nucleon channel. Here the relation in Eq. (7) is also satisfied, even though the signal is dominated by the 3/2− state. The results also show a similar mass for the 1/2− state and the 3/2− state, in accord with the experimental states of $N^*(1535)\frac{1}{2}^-$ and $N^*(1520)\frac{3}{2}^-$ which are close to each other.

Figure 3 presents results for the mass ratio extracted from the correlation functions for the $N^*(3/2^+)$ state to the nucleon ground state as a function of the mass ratio $(\pi/\rho)^2$. Mass ratios have minimal dependence on the uncertainties in determining the scale and the quark masses, so that a more reliable comparison with experiment can be made. Figure 4 shows the similar plot for the $N^*(3/2-)$ state. These ratios appear headed in the right direction compared to experiment.

3. Conclusion

We have obtained clear signals for spin-3/2 N* states on an anisotropic lattice with smeared operators and 100 configurations. States of both negative- and positive-parity are isolated with a parity projection technique. The need for spin projection is further demonstrated. The results for the pattern of the splittings appears consistent with experiment, but more study is needed to address systematic errors.

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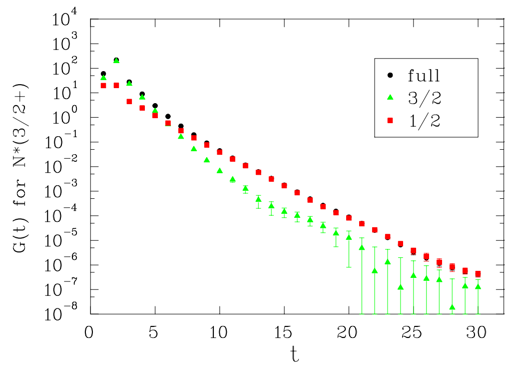

Figure 1. The various correlation functions for the positive-parity nucleon states at the smallest quark mass.

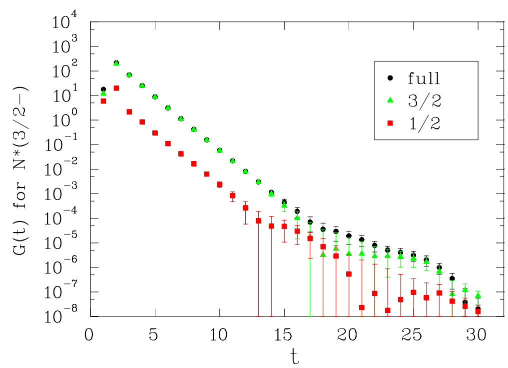

Figure 2. Similar to Figure 1, but for the negative-parity nucleon states.

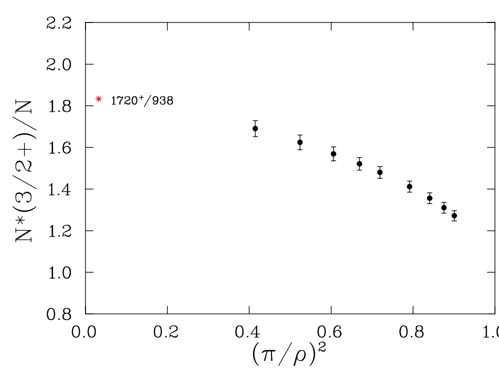

Figure 3. Mass ratio $N^*(3/2+)/N(1/2+)$ as a function of the mass ratio $(\pi/\rho)^2$. The experimental point is indicated for reference.

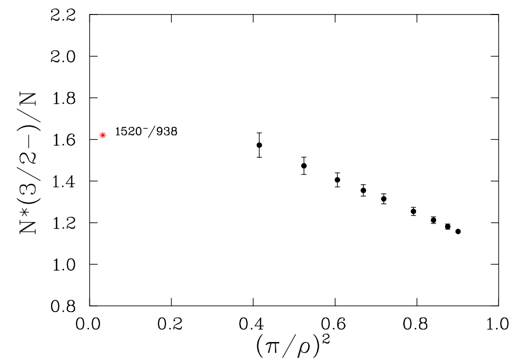

Figure 4. Similar to Figure 3, but for the ratio $N^*(3/2-)/N(1/2+)$.

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